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APPENDIX F

APPLICATION OF HISTORICAL FREQUENCY METHOD

The following problem illustrates the computational procedures involved in calculating the magnitude and frequency of occurrence of water levels at the State Pier, Providence, Rhode Island based on the formula and procedures discussed in Chapter 3. The basic and ordered annual peak water level data are given in Table F-1. The last two rows in the basic data, columns 3 and 4, shows historic water elevations of 18 feet for years 1635 and 1638. Historic data for these years are not considered a part of the more recent systematic record. By ignoring the historic record for the present, it is found by summing columns 4, 5, and 6 that:

$$\Sigma x = 269.1; \quad \Sigma x^2 = 1908.95$$

and

$$\Sigma x^3 = 16,352.259 \quad .$$

For the systematic record ($N = 43$) the mean, standard deviation, and skew coefficient according to Equations [3-6], [3-7b], and [3-8b] are:

$$\mu = \frac{\Sigma x}{N} = \frac{269.1}{43} = 6.2581$$

$$\sigma = \left[\frac{(\Sigma x^2) - \frac{(\Sigma x)^2}{N}}{N - 1} \right]^{1/2} = \left[\frac{1908.95 - \frac{(269.1)^2}{43}}{43 - 1} \right]^{1/2} = 2.3143$$

$$G = \frac{N^2 (\Sigma x^3) - 3N (\Sigma x) (\Sigma x^2) + 2 (\Sigma x)^3}{N (N - 1) (N - 2) \sigma^3}$$

$$= \frac{(43)^2 (16352.259) - 3(43)(269.1)(1908.95) + 2(269.1)^3}{43 (43 - 1) (43 - 2) (2.3143)^3} = 3.2053 \quad .$$

Use Equation [3-13] to test for high outlier.

$$\eta_H = \mu + K_N \sigma$$

TABLE F-1. ANNUAL PEAK WATER LEVELS, STATE PIER PROVIDENCE, R. I.

			BASIC DATA					ORDERED DATA					HISTORICAL DATA INFORMATION		
1	2	3	4	5	6	7	8	9	10	11	12	13			
MO	DAY	YEAR	x ELEV (FT)	x ²	x ³	RANK	YEAR	x ELEV (FT)	PLOTTING POSITION	$x - \bar{\mu}$	COLUMN 11 SQUARED	COLUMN 11 CUBED			
10	11	1931	4.2	17.64	74.088	1	1635	18.0	.0029	11.874	140.992	1555.251			
11	30	1932	5.3	28.09	148.877	2	1638	18.0	.0058	11.874	140.992	1555.251			
1	27	1933	6.0	36.00	216.000	3	1938	16.0	.0058	9.874	97.496	957.419			
1	16	1934	4.5	20.25	91.125	4	1954	14.9	.0220	8.774	76.983				
9	15	1935	4.3	18.49	79.507	5	1944	10.1	.0455	3.974	15.793				
10	1	1936	5.6	31.36	175.616	6	1960	8.0	.0691	1.874	3.512				
10	23	1937	5.0	25.00	125.000	7	1963	7.9	.0926	1.774	3.147				
9	21	1938	16.0	256.00	4096.000	8	1974	7.0	.1161	0.874	0.764				
4	2	1939	5.5	30.25	166.375	9	1950	7.0	.1397	0.874	0.764				
11	27	1940	5.2	27.04	140.608	10	1953	6.7	.1632	0.574	0.329				
5	11	1941	4.9	24.01	117.649	11	1942	6.6	.1867	0.474	0.225				
3	3	1942	6.6	43.56	287.496	12	1970	6.4	.2102	0.274	0.075				
3	6	1943	5.7	32.49	185.193	13	1972	6.3	.2338	0.174	0.030				
9	14	1944	10.1	102.01	1030.301	14	1966	6.3	.2573	0.174	0.030				
11	22	1945	6.3	39.69	250.047	15	1945	6.3	.2808	0.174	0.030				

TABLE F-1. ANNUAL PEAK WATER LEVELS, STATE PIER PROVIDENCE, R. I. (Con't)

BASIC DATA													ORDERED DATA				HISTORICAL DATA INFORMATION		
1	2	3	4	5	6	7	8	9	10	11	12	13							
MO	DAY	YEAR	x ELEV (FT)	x ²	x ³	RANK	YEAR	x ELEV (FT)	PLOTTING POSITION	$x - \bar{\mu}$	COLUMN 11 SQUARED	COLUMN 11 CUBED							
11	10	1946	5.0	25.00	125.000	16	1951	6.1	.3044	-0.026	0.001								
3	3	1947	5.9	34.81	205.379	17	1933	6.0	.3279	-0.126	0.016								
12	20	1948	5.1	26.01	132.651	18	1973	5.9	.3514	-0.226	0.021								
10	22	1949	5.6	31.36	175.616	19	1947	5.9	.3750	-0.226	0.021								
11	25	1950	7.0	49.00	343.000	20	1962	5.9	.3985	-0.226	0.021								
2	7	1951	6.1	37.21	226.981	21	1943	5.7	.4220	-0.426	0.181								
10	0	1952	4.8	23.04	110.592	22	1958	5.7	.4456	-0.426	0.181								
11	7	1953	6.7	44.89	300.763	23	1971	5.7	.4691	-0.426	0.181								
8	31	1954	14.9	222.01	3307.949	24	1968	5.6	.4926	-0.526	0.277								
10	16	1955	5.6	31.36	175.616	25	1936	5.6	.5162	-0.526	0.277								
3	16	1956	5.0	25.00	125.000	26	1955	5.6	.5397	-0.526	0.277								
2	15	1957	4.8	23.04	110.592	27	1949	5.6	.5632	-0.526	0.277								
4	3	1958	5.7	32.49	185.193	28	1964	5.5	.5867	-0.626	0.392								
12	29	1959	5.2	27.04	140.608	29	1939	5.5	.6103	-0.626	0.392								
9	12	1960	8.0	64.00	512.000	30	1975	5.4	.6338	-0.726	0.527								

TABLE F-1. ANNUAL PEAK WATER LEVELS, STATE PIER PROVIDENCE, R. I. (Con't)

BASIC DATA			ORDERED DATA					HISTORICAL DATA INFORMATION				
1	2	3	4	5	6	7	8	9	10	11	12	13
MO	DAY	YEAR	x ELEV (FT)	x ²	x ³	RANK	YEAR	x ELEV (FT)	PLOTTING POSITION	$x - \bar{\mu}$	COLUMN 11 SQUARED	COLUMN 11 CUBED
1	16	1961	5.3	31.36	148.877	31	1969	5.3	.6573	-0.826	0.682	
12	6	1962	5.9	34.81	205.379	32	1932	5.3	.6809	-0.826	0.682	
11	30	1963	7.9	62.41	493.039	33	1961	5.3	.7044	-0.826	0.682	
11	20	1964	5.5	30.25	166.375	34	1959	5.2	.7279	-0.926	0.857	
12	29	1966	6.3	39.69	250.047	35	1940	5.2	.7515	-0.926	0.857	
11	12	1968	5.6	31.36	175.616	36	1948	5.1	.7750	-1.026	1.053	
12	11	1969	5.3	28.09	148.877	37	1937	5.0	.7985	-1.126	1.268	
3	3	1970	6.4	40.96	262.144	38	1956	5.0	.8221	-1.126	1.268	
3	1	1971	5.7	32.49	185.193	39	1946	5.0	.8456	-1.126	1.268	
11	26	1972	6.3	39.69	250.047	40	1941	4.9	.8691	-1.226	1.503	
4	4	1973	5.9	34.81	205.379	41	1957	4.8	.8926	-1.326	1.758	
12	2	1974	7.0	49.00	343.000	42	1952	4.8	.9162	-1.326	1.758	
4	3	1975	5.4	29.16	157.464	43	1934	4.5	.9397	-1.626	2.643	
8	15	1635	18.0			44	1935	4.3	.9632	-1.826	3.334	
8	3	1638	18.0			45	1931	4.2	.9868	-1.926	3.709	

From Table 3-1, $K_N = 2.71$ for $N = 43$, thus

$$\eta_H = 6.2581 + (2.71)(2.3143) = 12.5 \text{ feet} .$$

A review of the systematic record reveals that the peak elevation of 16.0 feet in 1938 and the peak elevation of 14.9 feet in 1954 exceed the threshold value of 12.5 feet. Due to the fact that the systematic record is extended considerably as a result of the historic events, only the 1938 event will be used as a high outlier and the 1954 event will be considered as part of the systematic record. Because of the high outlier the statistics of the systematic record are re-evaluated as follows:

$$\Sigma x = 269.1 - 16 = 253.1$$

$$\Sigma x^2 = 1908.95 - 256 = 1652.95$$

$$\Sigma x^3 = 16,352.259 - 4096 = 12256.259 .$$

Thus for $N = 42$

$$\mu = \frac{\Sigma x}{N} = \frac{253.1}{42} = 6.0262$$

$$\sigma = \left[\frac{(\Sigma x^2) - \frac{(\Sigma x)^2}{N}}{N - 1} \right]^{1/2} = \left[\frac{1652.95 - \frac{(253.1)^2}{42}}{42 - 1} \right]^{1/2} = 1.7650$$

$$G = \frac{N^2 (\Sigma x^3) - 3N (\Sigma x) (\Sigma x^2) + 2(\Sigma x)^3}{N (N - 1) (N - 2) \sigma^3}$$

$$= \frac{(42)^2 (12256.259) - 3(42)(253.1)(1652.95) + 2(253.1)^3}{(42) (42 - 1) (42 - 2) (1.765)^3} = 3.5209 .$$

Adjustment of the statistics is required to account for the historic data including the high outlier. The weight factor W according to Equation [3-14] with a historic period from 1635 through 1975 or 341 years is:

$$W = \frac{H - Z}{N} = \frac{341 - 3}{42} = 8.0476 .$$

The sum of the historic water levels is:

$$\Sigma x' = 18 + 18 + 16 = 52 .$$

By Equation [3-15]

$$\bar{\mu} = \frac{W (\Sigma x) + \Sigma x'}{H} = \frac{(8.0476)(253.1) + 52}{341} = 6.126 .$$

Based on the adjusted mean, $x - \bar{\mu}$ is computed as shown in column 11, $(x - \bar{\mu})^2$ in column 12 and $(x' - \bar{\mu})^3$ in column 13.

The sums are:

$$\Sigma (x - \bar{\mu})^2 = 128.03$$

$$\Sigma (x' - \bar{\mu})^2 = 379.48$$

$$\Sigma (x' - \bar{\mu})^3 = 4310.949 .$$

By use of Equation [3-16] it is found that

$$\begin{aligned} \bar{\sigma} &= \left[\frac{W \Sigma (x - \bar{\mu})^2 + \Sigma (x' - \bar{\mu})^2}{H - 1} \right]^{1/2} = \left[\frac{(8.0476)(128.03) + 379.48}{341 - 1} \right]^{1/2} \\ &= 2.0363 . \end{aligned}$$

From Equation [3-17]

$$\begin{aligned} \bar{G} &= \frac{H}{(H - 1)(H - 2) \sigma^3} \left[\frac{W(N - 1)(N - 2) \sigma^3 G}{N} \right. \\ &\quad \left. + 3W (N - 1) (\mu - \bar{\mu}) \sigma^2 + WN (\mu - \bar{\mu})^3 + \Sigma (x' - \bar{\mu})^3 \right] \\ \bar{G} &= \frac{341}{(340)(339)(2.0363)^3} \left[\frac{(8.0476)(41)(40)(1.765)^3 (3.5209)}{42} \right] \end{aligned}$$

$$+ 3(8.0476)(41)(6.0262 - 6.126)(1.765)^2$$

$$+ (8.0476)(42)(6.0262 - 6.126)^3 + 4310.949 \Big] .$$

$$\bar{G} = 3.5341, \text{ say } 3.5 .$$

Table F-2 shows summaries of the exceedence frequency curve. The first column is a tabulation of the prescribed exceedence probabilities and column 2 shows the corresponding frequency factors for $\bar{G} = 3.5$. (The K values can be found in Bulletin No. 17b.) Column 3 shows the frequency factor times the standard deviation and column 4 is the solution of Equation [3-18], or

$$\eta = \bar{\mu} + \bar{\sigma}K .$$

The upper and lower confidence limits for levels of significance of .05 and .95 are computed as follows:

The standard normal deviate t at confidence level 0.05 is 1.64485 as found in Bulletin No. 17b for zero skew. From Equations [3-26] and [3-27] it is found that

$$a = 1 - \frac{t^2}{2(N - 1)} = 1 - \frac{(1.64485)^2}{2(42 - 1)} = 0.967$$

$$b = K^2 - \frac{t^2}{N} = K^2 - \frac{(1.64485)^2}{42} = K^2 - .0644174172 .$$

The values of K^2 are shown in column 5 in Table F-2 and the values of b are shown in column 6. The solutions of Equation [3-24] and [3-25] are shown in columns 9 and 10. The upper and lower confidence limits for the water levels according to Equations [3-22] and [3-23] are shown in columns 11 and 12, respectively. The water level elevations corresponding to the expected probabilities are shown in column 13. These are determined by using Equation

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TABLE F-2. FREQUENCY CURVE AND CONFIDENCE LIMITS

EXCEEDENCE FREQUENCY CURVE				CONFIDENCE LIMITS									
1	2	3	4	5	6	7	8	9	10	11	12	13	
P	K	$K\bar{\sigma}$	$\eta(\text{FT})$	K^2	b	ab	$(K^2 - ab)$	$\frac{K_1}{K+Col.8}$	$\frac{K_1}{K+Col.8}$	$\frac{K_1}{K+Col.8}$	$\eta(\text{FT})$.96 LIMIT	$\eta(\text{FT})$.05 LIMIT	$\eta(\text{FT})$ EXPECTED PROBABILITY
.002	6.64627	13.5337	19.5	44.1729	44.1085	42.6529	1.2329	8.1481	5.5981	22.6	17.5	21.7	
.005	5.25291	10.5965	16.8	27.5930	27.5286	26.6202	0.9863	6.4521	4.4122	19.2	15.1	18.2	
.010	4.22473	8.6028	14.7	17.8483	17.7839	17.1970	0.8071	5.2035	3.5343	16.7	13.3	15.7	
.020	3.22641	6.5699	12.7	10.4097	10.3433	10.0020	0.6385	3.9968	2.6762	14.2	11.6	13.3	
.040	2.26862	4.6120	10.7	5.1466	5.0822	4.9145	0.4818	2.8443	1.8480	11.9	9.9	11.1	
.100	1.09552	2.2308	8.4	1.2002	1.1358	1.0983	0.3192	1.4630	0.8028	9.6	7.8	8.5	
.200	0.32171	0.6551	6.8	0.1035	0.0391	0.0378	0.2563	0.5977	0.0676	7.4	6.3	6.8	
.500	-0.41253	-0.8400	5.3	0.1702	0.1058	0.1023	0.2606	-0.1571	-0.6961	5.8	4.7	5.3	
.800	-0.56242	-1.1453	5.0	0.3163	0.2519	0.2436	0.2696	-0.3028	-0.8604	5.5	4.4	5.0	
.900	-0.57035	-1.1614	5.0	0.3253	0.2620	0.2534	0.2681	-0.3126	-0.8671	5.5	4.4	5.0	
.950	-0.57130	-1.1633	5.0	0.3264	0.2620	0.2534	0.2702	-0.3114	-0.8702	5.5	4.4	5.0	
.990	-0.57143	-1.1636	5.0	0.3265	0.2621	0.2535	0.2702	-0.3115	-0.8704	5.5	4.4	5.0	

NOTE: $\bar{G} = 3.5$

$\bar{\sigma} = 2.0363$

$\bar{\mu} = 6.126$

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[3-28a] through [3-28f], interpolation of the intermediate expected probabilities and plotting the resulting frequency curve. Figure F-1 shows the expected probability curve derived together with the observed peak water levels.

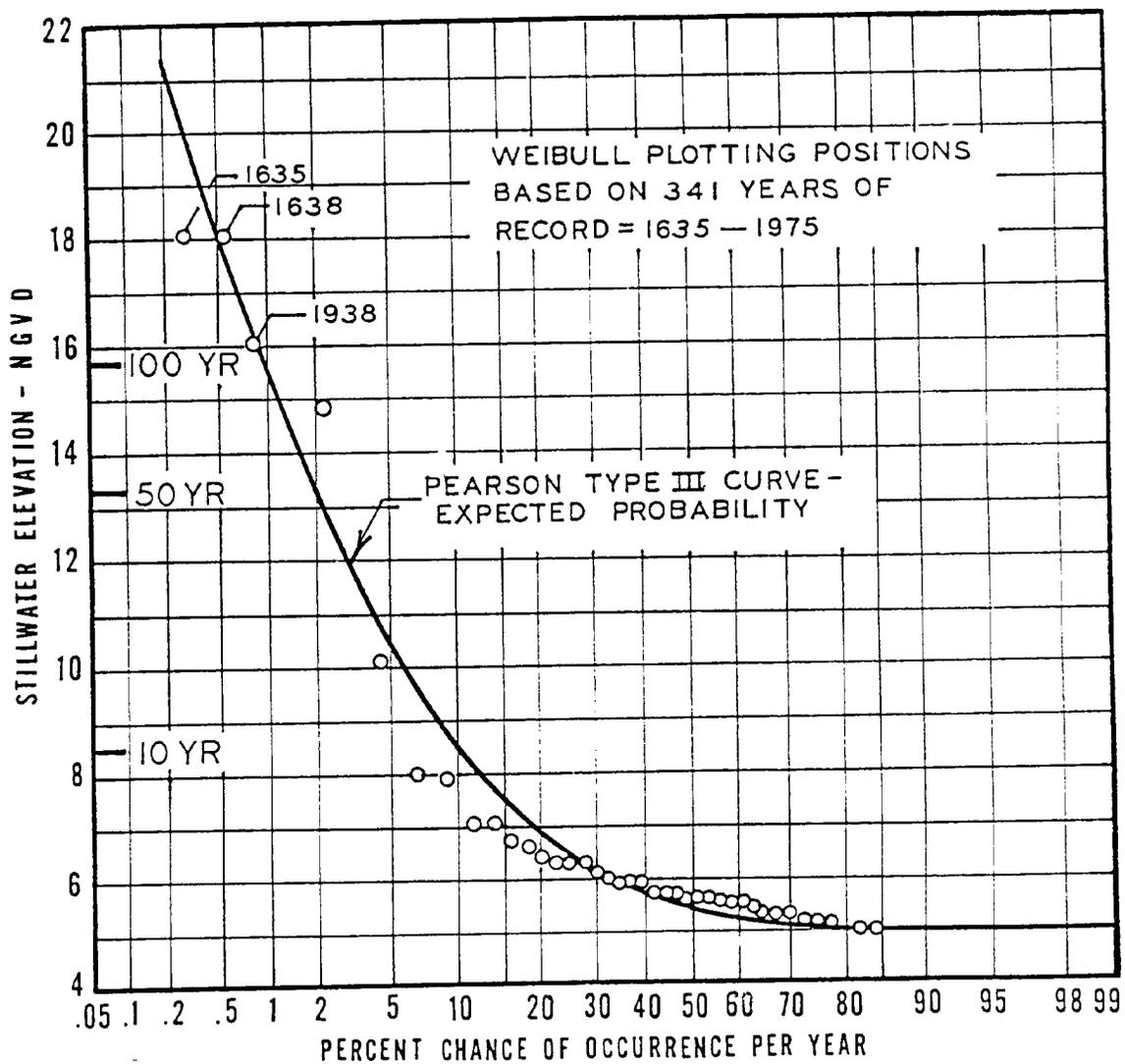


Figure F-1. Frequency of water levels at the State Pier, Providence, R. I.